Using Research to Investigate and Enhance Learning in Upper-division Mechanics

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Typical content in upper-division mechanics

Foundational topics
(introductory level)

- Vectors
- Kinematics
- Newton's laws
- Work, energy, energy conservation
- Linear and angular momentum

New applications and extensions

- Velocity-dependent forces
- Linear and non-linear oscillations
- Conservative force fields
- Non-inertial reference frames
- Central forces, Kepler's laws

New formalism and representations

- Scalar and vector fields; del operator; gradient, curl
- Variational methods; Lagrangian mechanics
- Phase space diagrams

What makes upper division mechanics interesting to *teach*

- *Content and methodology:* Students can investigate and model physical systems in more sophisticated ways
  - Higher level mathematics
  - Computational tools
- *Population:* Students are predominantly physics majors (and minors)
- *Assessment:* Small class size allows more in-depth probing of conceptual understanding and problem-solving skills

What makes upper division mechanics interesting for *physics education research*

- To what extent have students already acquired a functional understanding of foundational topics?
  - To what extent do results from PER at the *introductory* level predict difficulties experienced by *advanced* students?
  - What *unexpected* things are students doing as they encounter new topics in upper level mechanics?
- How is the use of mathematics different from that at the introductory level?

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- To what extent have students already acquired a functional understanding of foundational topics?
  - To what extent do results from PER at the *introductory* level predict difficulties experienced by *advanced* students?
  - What *unexpected* things are students doing as they encounter new topics in upper level mechanics?
- How is the use of mathematics different from that at the introductory level?

Take-home message: Conceptual understanding **must** be an essential focus in upper level mechanics.

Reason #1: Many conceptual and reasoning difficulties *persist* beyond introductory level

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At the introductory level, students have difficulty discriminating between a **quantity** and its **rate of change:**

- position *vs.* velocity*
- velocity *vs.* acceleration*
- height *vs.* slope of a graph**
- electric field *vs.* electric potential †
- electric (or magnetic) flux *vs.* change in flux
- *…and many other examples*

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* Trowbridge and McDermott, Am. J. Phys. **48** (1980) and **49** (1981);
Flores and Kanim, Am. J. Phys. **72** (2004); Shaffer and McDermott, Am. J. Phys. **73** (2005).

** McDermott, Rosenquist, and van Zee, Am. J. Phys. **55** (1987).

† Allain, Ph.D. dissertation, NCSU, 2001; Maloney *et al.,* Am. J. Phys. Suppl. **69** (2001).

What we teach about conservative forces

A force $\vec{F}(\vec{r})$ is conservative if and only if:

- the work by that force around any closed path is zero
- $\vec{\nabla} \times \vec{F} = 0$ at all locations
- a potential energy function $U(\vec{r})$ exists so that $\vec{F} = -\vec{\nabla} U$

(generalization of $\vec{E} = -\vec{\nabla} V$ from electrostatics)

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Research question: What difficulties do students have in understanding and applying this relationship?

"Equipotential map" pretest

Intermediate mechanics

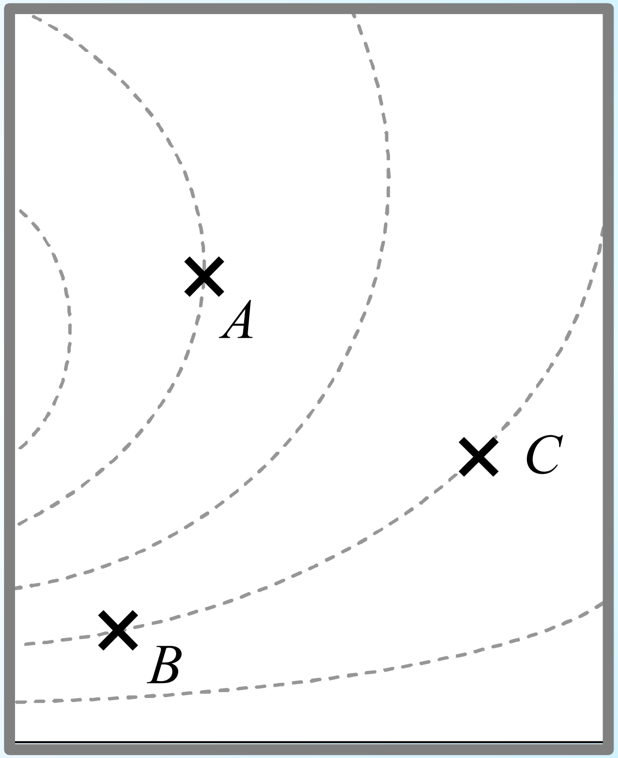
After all lecture instruction in introductory E&M

In the region of space depicted at right, the dashed curves indicate locations of *equal potential energy* for a test charge $+q_{\text{test}}$ placed within this region.

It is known that the potential energy at location *A* is *greater than* that at *B* and *C*.

A. At each location, draw an arrow to indicate the direction in which the test charge $+q_{\text{test}}$ would move when released from that location. Explain.

B. Rank the locations *A, B,* and *C* according to the magnitude of the force exerted on the test charge $+q_{\text{test}}$. Explain your reasoning.



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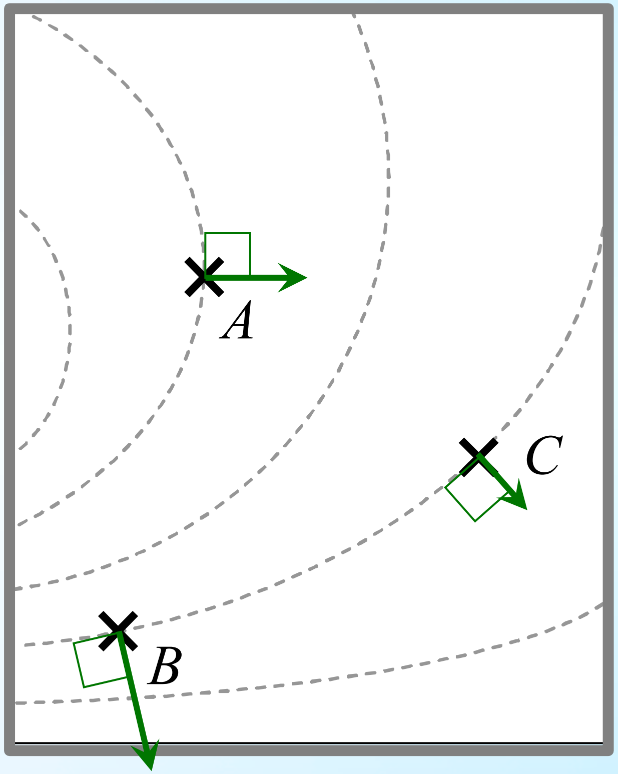


(Qualitatively correct force vectors are shown.)

Equipotential map pretest: Results

Intermediate mechanics, GVSU (*N* = 73, 8 classes)

After all lecture instruction in introductory E&M

Percent correct *with correct reasoning:*
(rounded to nearest 5%)

Part A (Directions of force vectors)	50%	(35/73)
Part B (Ranking force magnitudes)	20%	(14/73)
Both parts correct	**15%**	**(9/73)**

Similar results have been found after lecture instruction at U. Maine and pilot test sites (N = 115, 11 classes).

Equipotential map pretest: Results

Intermediate mechanics

After all lecture instruction in introductory E&M

Most common *incorrect* ranking: $F_A > F_B = F_C$

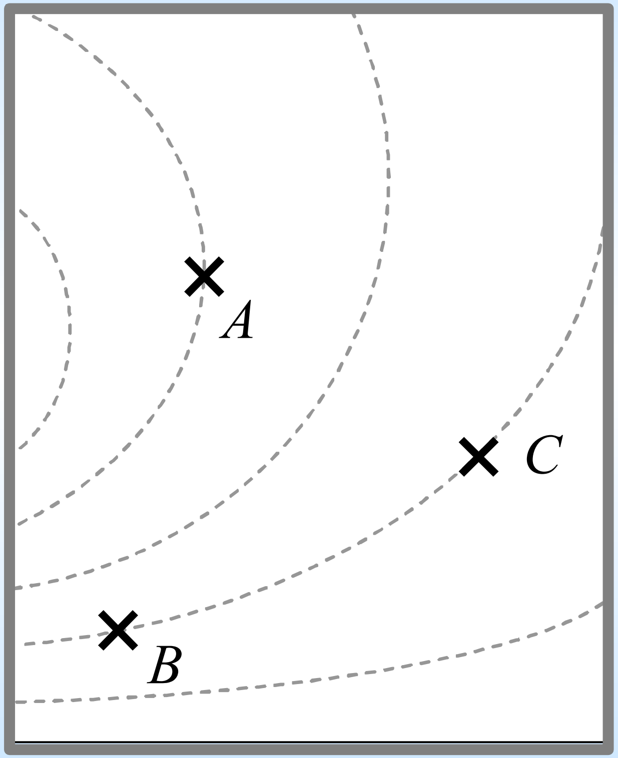
Example: "Since F is proportional to V, higher V means higher F."

Example:

"[$V_A > V_B = V_C$] … $F(x) = - dV/dx$

$\therefore$ $F_C = F_B$ in magnitude and

$F_A > F_C$ in magnitude."



Failure to discriminate between a quantity *(potential energy U)* ***and its rate of change*** *(force $\vec{F} = -\vec{\nabla} U$)*

Reason #2: Conceptual and mathematical difficulties are often *intertwined*

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What we teach about harmonic oscillators:

	Equation of motion	Solution for *x(t)*
Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(\omega_o t + \varphi)$ where $\omega_o = \sqrt{k/m}$
Underdamped motion $(\gamma < \omega_o)$	$m\ddot{x} = -kx - c\dot{x}$ $\left(\ddot{x} = -\omega_o^2 x - 2\gamma\dot{x}\right)$	$x(t) = A_o\, e^{-\gamma t} \cos(\omega_d t + \varphi)$ where $\omega_d = \sqrt{\omega_o^2 - \gamma^2}$

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$\Rightarrow$ Frequency depends on mass and spring constant

$\Rightarrow$ Amplitude has no effect on frequency or period

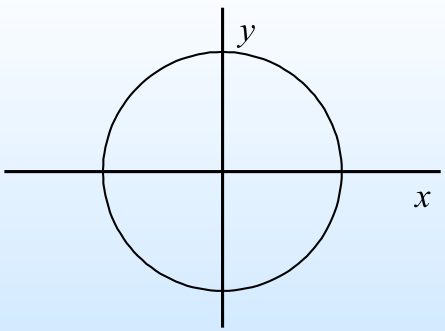
2D oscillator pretest

Consider the motion of a 2D oscillator, with $U(x, y) = ½\, k_1x^2 + ½\, k_2y^2$, or equivalently, $U(x, y) = ½\, m\omega_1^2x^2 + ½\, m\omega_2^2y^2$.

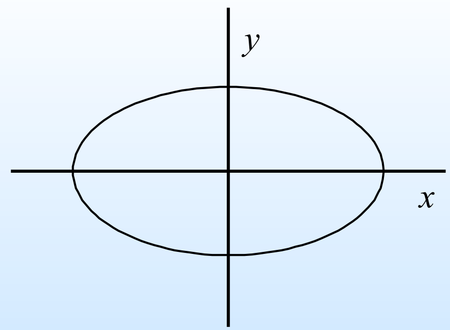
Q: For each *x-y* trajectory shown, could the oscillator follow that trajectory?

If so: Is $\boldsymbol{\omega_1}$ *greater than, less than,* or *equal to* $\boldsymbol{\omega_2}$*?* Explain.*

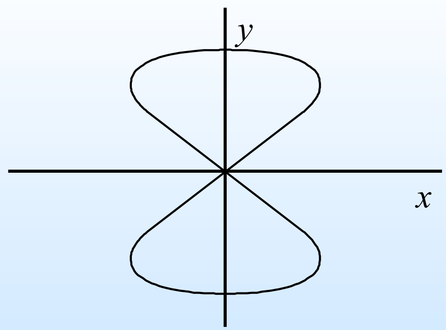
If not: Explain why not.



Case #1



Case #2



Case #3

(Ans: $\omega_1 = \omega_2$***)***

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(Ans: $\omega_1 > \omega_2$***)***

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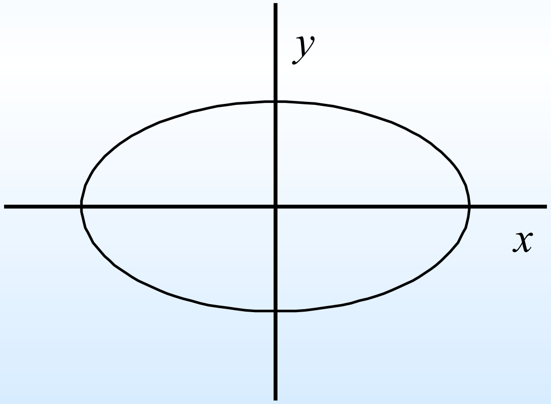
* Original phrasing asked for a comparison between $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$.

2D oscillator pretest: Results

Intermediate mechanics, GVSU (4 classes) and UME (1 class)

After relevant lecture instruction

- Few students **(0% - 15%)** answered all cases correctly.
- Most incorrect responses based on compensation arguments* involving relative amplitudes along *x*- and *y*-axes:



Case #2

Example responses for Case #2:

"$k_1 < k_2$, the spring goes farther in the *x*-direction, so spring must be less stiff in that direction."

"$\omega_2 > \omega_1$. Since we now have an oval curve with the *x*-axis longer, ω_2 must be greater to compensate."

* R.A. Lawson and L.C. McDermott, *Am. J. Phys.* **55** (1987); O'Brien Pride, Vokos, and McDermott, *Am. J. Phys.* **66** (1998).

Alternate version of 2D oscillator pretest

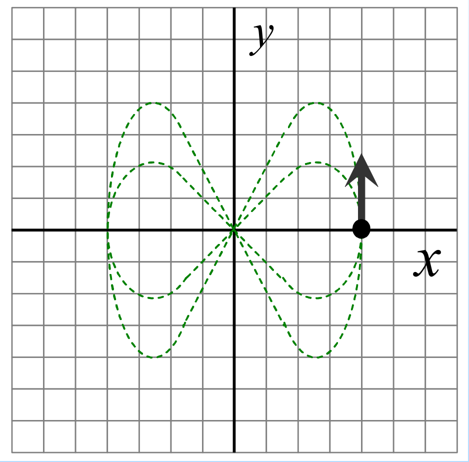
Consider an object that moves along a horizontal frictionless surface (*e.g.*, an air hockey puck on a level air table). Suppose that the object moves under the influence of a net force expressed as follows:

$$\mathbf{F}_{\mathbf{net}}\left(x,y\right) = \left(-k_x x\,\hat{i}\right) + \left(-k_y y\,\hat{j}\right)$$

Note: The above net force can be modeled by two long, mutually perpendicular springs with force constants k_x and k_y.

Q: For each case, carefully sketch a qualitatively correct *x*-*y* trajectory for the object. Explain your reasoning.

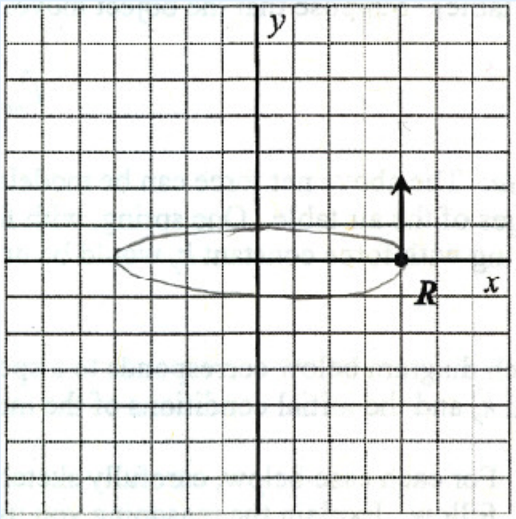
Example non-isotropic case, $\boldsymbol{k_y = 4k_x}$*:*



Alternate 2D oscillator pretest: Results

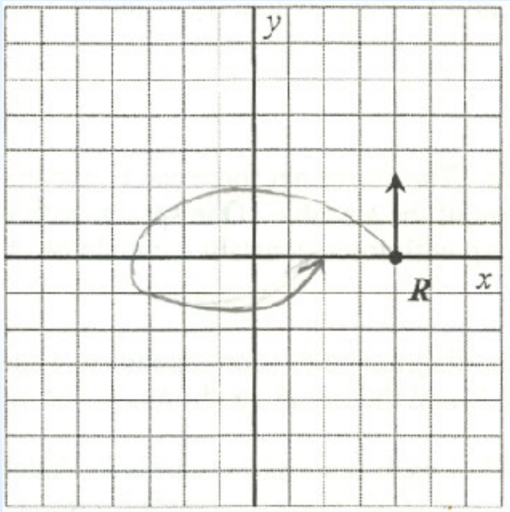
GVSU (2 classes), UNH (1 class)

"Compensation arguments" with amplitudes and force constants:



$k_y = 4k_x$

"An ellipse rather than a circle because the spring forces are different."



$k_y = 4k_x$

"The object travels less in the *y*-direction because of the stiffer spring. The springs attempt to return the object to equilibrium."

Reason #2: Conceptual and mathematical difficulties are often *intertwined*

What we teach about harmonic oscillators:

	Equation of motion	Solution for *x(t)*
Simple harmonic motion	$m\ddot{x} = -kx$	$x(t) = A_o \cos(\omega_o t + \varphi)$ where $\omega_o = \sqrt{k/m}$
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$\Rightarrow$ Damping force lowers oscillation frequency ($\boldsymbol{\omega_d} < \boldsymbol{\omega_o}$)

$\Rightarrow$ Damping force causes amplitude to decrease over time, with *constant* ratio between successive maxima

"Underdamped oscillator" pretest

(excerpt)

A simple harmonic oscillator is released from rest at $x = +\ 1.00$ m.

The oscillator is set into motion again from the same location, except now with a retarding force that is linear with respect to velocity. The oscillator now reaches $x = +\ 0.80$ m after one period.

A. During the first full oscillation of motion, is it possible to determine what fraction of the oscillator's total energy was dissipated?

Ans.: $1 - (.80/1.00)^2 =$ **9/25**, or **36%**

B. When the oscillator finishes a *second* full oscillation, is it possible to predict the maximum displacement of the oscillator?

Ans.: $(1.00\ \text{m})\bullet(.80/1.00)^2 =$ **0.64 m**

For each question, <u>either</u> determine the answer (if possible) <u>or</u> explain what additional information you need to find it.

Underdamped oscillator pretest: Results

After lectures, GVSU (1 class), SPU (1 class), and WCU (4 classes)

- Only ~ **50%** of students correctly applied position dependence of potential energy ($U(x) = \frac{1}{2}kx^2 \propto x^2$):

  Examples of incorrect responses for part A:

  "We need the mass and spring constant."

  "If 20% of the amplitude is lost, then one can deduce that 20% of the energy is lost."

- Only ~ **35%** of students correctly recognized that the ratio of successive maxima is constant:

  Example incorrect response for part B:

  "Max. displacement after two cycles is x = 0.60 m [not x = 0.64 m] because the retarding force is linear."

Reason #3: Specific conceptual and reasoning difficulties must be *directly* addressed

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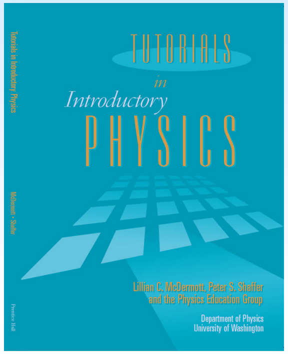
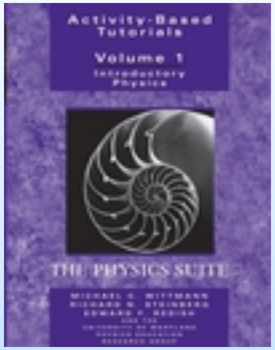


A research-tested guided-inquiry approach for supplementing lectures in *introductory physics:*

- Teaching-by-questioning strategies designed to:
  - address specific conceptual and reasoning difficulties
  - help students connect the mathematics to physics



- Tutorial components:
  - pretests (ungraded quizzes, ~10 min)
  - tutorial worksheets (small-group activities, ~50 min)
  - tutorial homework
  - examination questions (post-tests)

*Intermediate Mechanics Tutorials**

Collaboration between GVSU (Ambrose) and UME (Wittmann)

- Newton's laws and velocity-dependent forces
- Simple harmonic motion
- Damped harmonic motion
- Driven harmonic motion
- Phase space diagrams
- Conservative force fields
- Harmonic motion in two dimensions
- Accelerating reference frames
- Orbital mechanics
- Generalized coordinates and Lagrangian mechanics

* Development and dissemination support by NSF grants DUE-0441426 and DUE-0442388

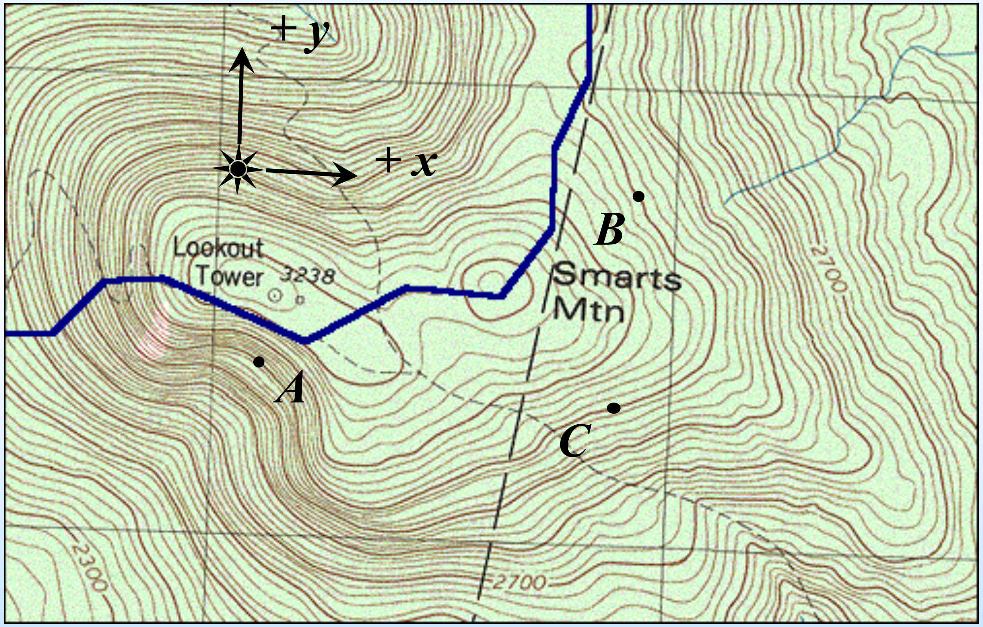
Helping students connect meaning between the physics and the mathematics

In the tutorial *Conservative forces and equipotential diagrams:*

Students develop a qualitative relationship between **force vectors** and local **equipotential contours**…

…and construct an **operational definition of the gradient** of potential energy:

$$\vec{\nabla} U = \left(\frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} \right)$$



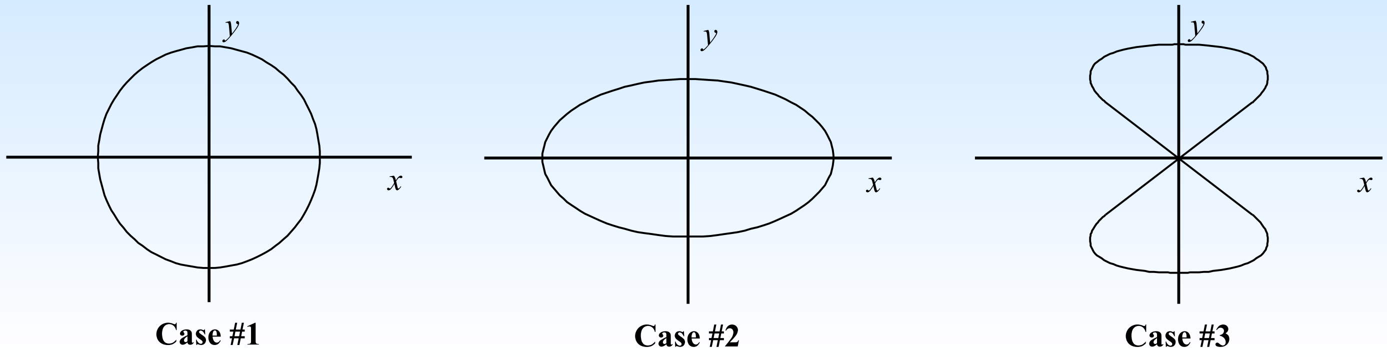
Helping students connect meaning between the physics and the mathematics

Tutorial concludes with students reflecting upon what gradient *means* **<u>and</u>** what it *does not mean:*

Summarize your results: Does $\vec{\nabla} U$ ...

- point in the direction of *increasing* or *decreasing* potential energy?
- point in the direction in which potential energy changes the *most* or the *least* with respect to position?
- **have the *same magnitude* at all locations having the *same potential energy?* Explain why or why not.**

Helping students build and refine productive intuitions about the physics



In the tutorial *Harmonic motion in two dimensions,* students are guided to recognize:

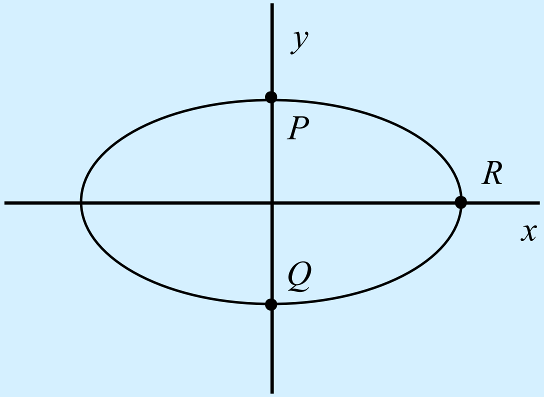
- how many oscillations occur along the *y*-axis for each oscillation along the *x*-axis
- how differences in force constants affect periods and frequencies
- how phase difference between *x*- and *y*-motions affect trajectories of isotropic oscillators

Students are guided to connect amplitude to potential energy (not frequency)

Excerpt from tutorial homework—revised in 2003—from *Harmonic motion in two dimensions:*

A. Critique the following statement. Explain.

> "The oscillator goes farther in the *x*-direction than in the *y*−direction. That means the spring in the *y*-direction must be stiffer than the spring in the *x*-direction."



B. Rank points *P, Q,* and *R* according to (i) total energy, (ii) potential energy, (iii) kinetic energy.

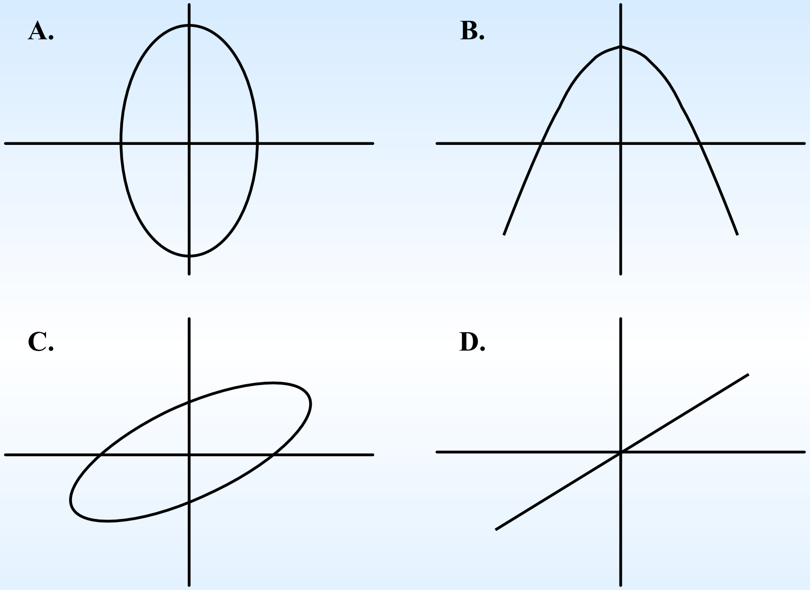
Explain how the difference in the *x*- and *y*-amplitudes, used *incorrectly* in the statement in part A, can help justify a *correct* answer here in part B.

Examples of assessment questions

On written exams after modified instruction (GVSU)

Qualitative: "Is k_x *greater than, less than,* or *equal to* k_y? Explain."

Quantitative: "Evaluate the ratio k_y/k_x. Show all work."

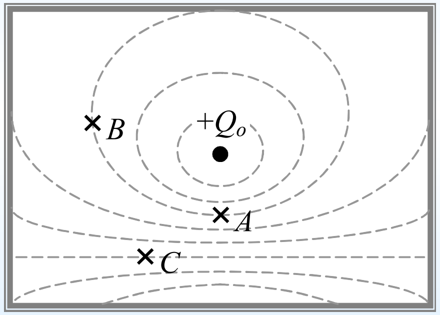


Before revised tutorial HW ('01 – '02): ≈ 50% correct

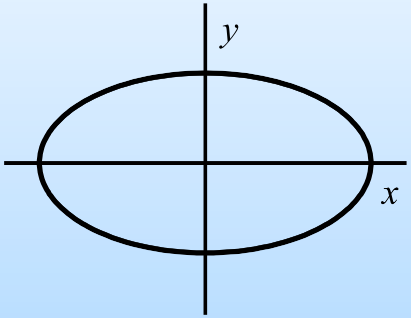
After revised tutorial HW ('03 – present): ≈ **90% correct**

Summary and reflections

- Physics majors in *advanced* courses can and do experience conceptual and reasoning difficulties similar in nature to those already identified at the *introductory* level.
  - Difficulty discriminating between a **quantity** and its **rate of change**



  - Reliance on inappropriate **“compensation arguments”** with two or more variables



Summary and reflections

- Students need guidance to extract physical meaning from the mathematics.
  - **Guided sense-making** seems more important than derivations.
  - Students need practice articulating **in their own words** the physical meaning expressed in the *graphical representations* and in the *mathematics* they use.
- Specific difficulties must be addressed *explicitly* and *repeatedly* for meaningful learning to occur.
  - Assessments of **conceptual underpinnings** should be done explicitly and repeatedly.

Intermediate Mechanics Tutorials

Project website:

http://faculty.gvsu.edu/ambroseb/research/IMT.html

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Summary and reflections

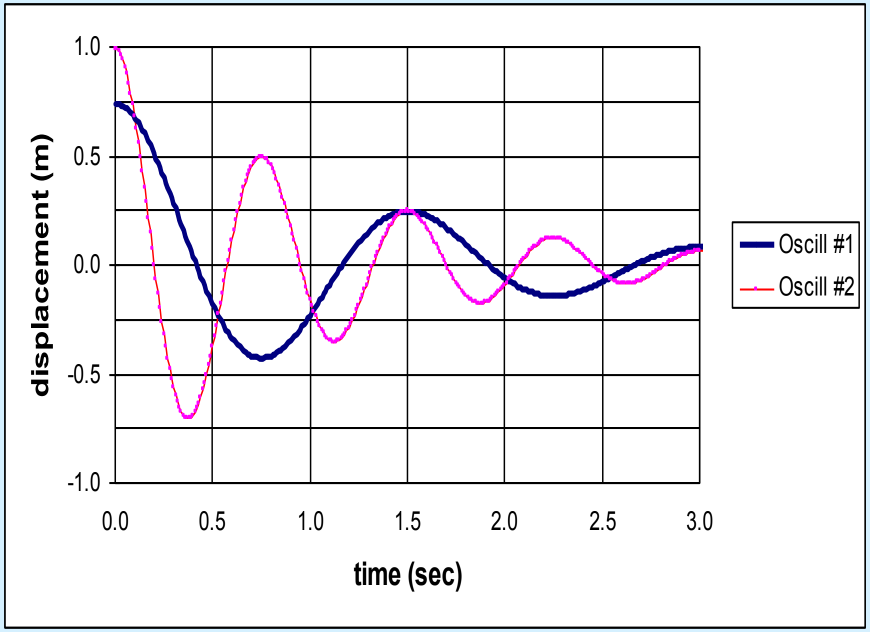
- Intermediate mechanics offers rich opportunities for exploring how students navigate the interplay between math and physics.

Q: Which oscillator, if any, has:

- the larger *damping constant (γ)?*
- the larger *quality factor?*

Q: Use the graph for oscillator #1 (blue) to deduce values of *a* and *b:*

$$\ddot{x} + a\dot{x} + b = 0$$

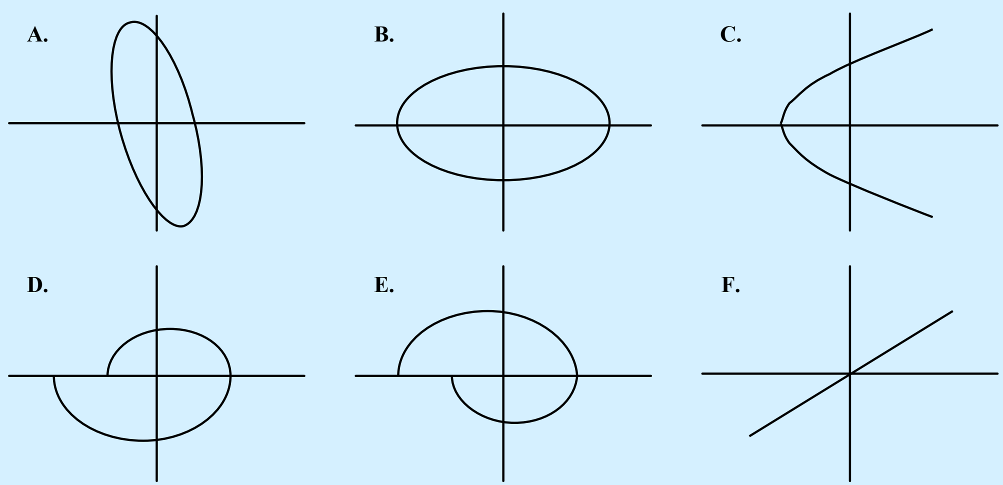


Summary and reflections

- Intermediate mechanics also offers context in which to assess coherence and organization of student knowledge.

Identify which diagram(s), if any, could be:

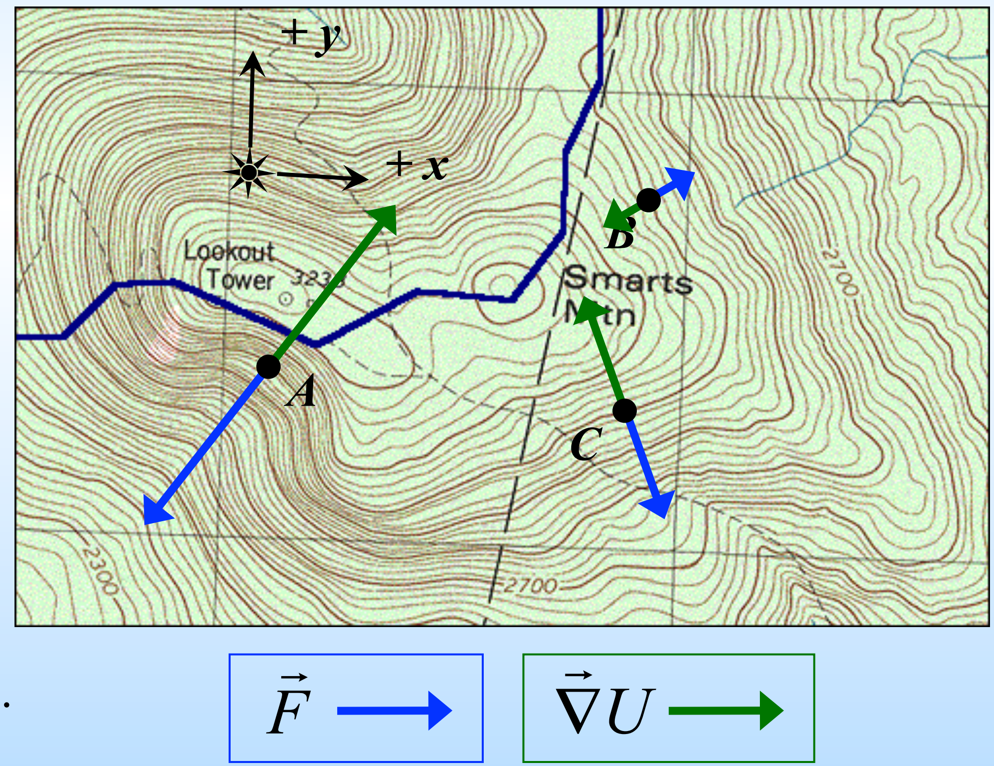
i. phase space plot of a simple harmonic oscillator
ii. phase space plot of an underdamped oscillator
iii. trajectory of a 2-D oscillator for which $k_y > k_x$
iv. trajectory of a 2-D oscillator for which $k_y = k_x$



Helping students connect meaning between the physics and the mathematics

Students construct operational definition of *gradient:*

- *In words,* how would you calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$?
- Is $\frac{\partial U}{\partial x}$ pos, neg, or zero?
- Is $\frac{\partial U}{\partial y}$ pos, neg, or zero?
- Compare $\left|\frac{\partial U}{\partial x}\right|$ and $\left|\frac{\partial U}{\partial y}\right|$.
- Draw $\vec{\nabla} U = \left(\frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} \right)$.



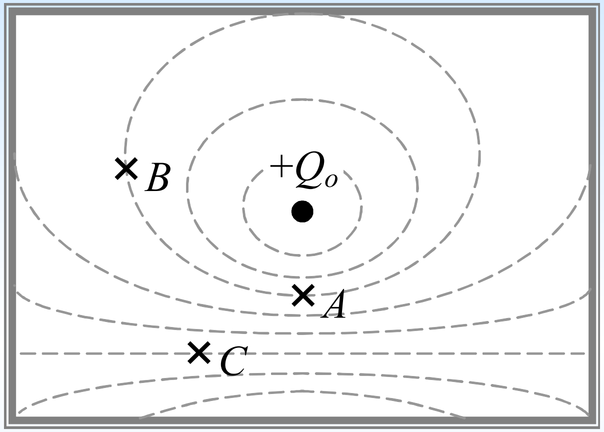
Examples of assessment questions

On written exams after modified instruction

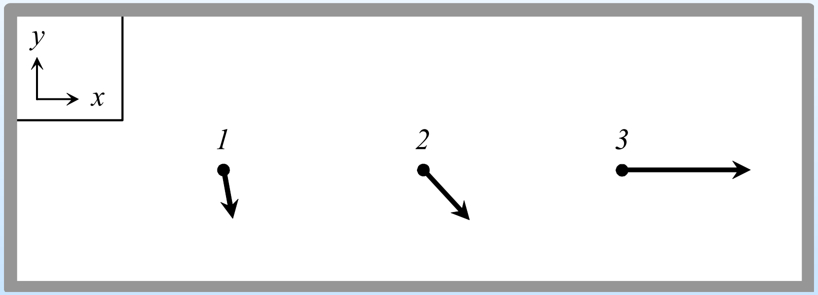
Task: Given equipotential map, predict directions and relative magnitudes of forces.

GVSU: **20/23 correct** (2 classes)

SPU: **8/11 correct** (1 class)



Task: Given several force vectors, sketch possible equipotential map and rank points by potential energy.



GVSU: **14/30 correct** (3 classes)